EFFECT OF TEMPERATURE REGIMES ON THE STABILITY OF THE PROCESS OF DRAWING OPTICAL FIBERS

V. L. Kolpashchikov, Yu. I. Lanin,
O. G. Martynenko, and A. I. Shnip

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1. The drawing of fibers from melts (or solutions) and their subsequent solidification is a widely used production process. However, it has become necessary to conduct this process with a high degree of stability only since the advent of optical fibers. By stability here, we mean that the parameters of the process should be such that the fluctuations of a fiber having an outlet section with a diameter on the order of 100 μm should not exceed approximately 1 μm. This means that it is necessary not only to eliminate the internal instabilities inherent in the drawing operation [1-5], but also to minimize the effect of unavoidable fluctuations in regime parameters on the diameter of the fiber being drawn. Thus, it is obviously important to theoretically analyze the stability of the drawing operation and its reaction to external disturbances.

These topics have been examined by several authors for the isothermal case of drawing. It has been found that in this case the internal instability of the drawing process increases when the drawing coefficient $W = \ln (v_d/v_f)$ (v_d and v_f are the drawing rate and the feed of the semifinished product) exceeds a certain value $W \approx 3$. This leads to a change from a steady regime to a regime of nonlinear oscillations [1]. Nevertheless, commercial drawing operations, including the drawing of optical fibers, proceed successfully with a high degree of stability at values of the draft coefficient on the order of 10-11. The reason for this discrepancy is evidently that the conditions of drawing of optical fibers are quite different from the conditions on which the above-mentioned studies were based. This applies particularly to the nonisothermal nature of the process, when the viscosity of the melt is heavily dependent on the temperature. Study of the stability of the drawing operation with allowance for additional factors – especially the variability of viscosity – was undertaken in [5, 6]. However, there has been almost no investigation of how stability is affected by the nonisothermality typical of the drawing of optical fibers or of the dependence of the stability of the process and its reaction to perturbations on the temperature regimes.

The goal of the present investigation is to analyze the dependence of the stability of the drawing process and its reaction to external perturbations on temperature regimes in the deformation zone under strongly nonisothermal conditions characteristic of optical fiber drawing and with allowance for the actual temperature dependence of the viscosity of quartz glass. This dependence is usually approximated by the Taman–Felcher formula [7]. Using a quantity which is the inverse of the Trouton viscosity* – fluidity μ^{-1} – we obtain the following from this formula

$$\mu^{-1}(T) = \begin{cases} \mu_0 e^{-\frac{T_0}{T-T_-}}, & T \geqslant T_-, \\ 0, & T < T_-. \end{cases} \quad (1.1)$$

Analysis of experimental data [7-9] gives the following values for the constants in this formula: $\mu_0^{-1} = 49.38\ \mathrm{Pa^{-1}\cdot sec^{-1}}$, $T_0 = 25.417$ K, $T_- = 673$ K.

2. The process of drawing an optical fiber involves the winding of a thin filament of quartz glass melted in a heater about a receiving drum. The fiber is drawn from a cylindrical semifinished product in the form of a rod. The rod is fed continuously into the heating zone, and drawing occurs as a result of tension of the fiber created by rotation of the drum. A

*The Trouton viscosity is three times greater than the shear viscosity of the fluid and is numerically equal to the unit tensile force in the axial tension of a cylindrical element of the fluid with a unit rate of relative elongation.

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mathematical model of the process should describe the hydrodynamics of a stream of a viscous Newtonian fluid with a length considerably greater than its radius and a vriable fluidity, the stream flowing under the action of a tensile force. There is no need for the model to account for forces associated with inertia, surface tension, gravity, or the resistance of the surrounding gas. According to estimates similar to those made in [10] for typical conditions in the drawing of a quartz fiber, all of these forces amount to no more than tenths of a percent of the viscous forces. Thus, viscous forces are predominant. Also, the intensive heat radiation which takes place throughout the volume of the stream at temperatures above 2000 deg C makes it possible to consider the temperature to be uniform across the stream and to assume that the temperature distribution along the stream axis coincides with the profile temperature distribution in the heater.

With allowance for these circumstances, we can describe the drawing process with a unidimensional hydrodynamic model which includes equations of momentum and mass conservation [10]:

$$sv_z = \mu^{-1}[T(z)]F(\tau); \tag{2.1}$$

$$s_z v + sv_z + s_\tau = 0. \tag{2.2}$$

Here, z is the coordinate directed along the stream axis, with the origin at the point of the temperature maximum; τ is time; $v(z, \tau)$ is the velocity of the stream (at point z at the moment of time τ); $s(z, \tau)$ is the cross-sectional area of the stream; $F(\tau)$ is the drawing force; $T(z)$ is the longitudinal temperature distribution in the heater.

To be able to model different temperature regimes, we will approximate the temperature distribution in the heater with a fourth-order polynomial

$$T(z) = \frac{(z-a)(z-b)(z-c)(z-d)}{abcd}(T_+ - T_-) + T_-, \quad a<b<0<c<d. \tag{2.3}$$

Equation (2.3) and, thus, Eqs. (2.1) and (2.2) are examined in the interval $z \in [b, c]$ because, in accordance with (1.1), fluidity vanishes outside this interval and thus points b and c are the boundaries of the deformation zone; here, the maximum temperature, equal to T_+, is always reached at the point z = 0.

Introducing the function $y(z, \tau)$, connected with s and v by the relations

$$s = s_0 y_z, \quad v = -y_\tau / y_z,$$

where s_0 is the time-averaged inlet cross section, we reduce system of equations (2.1-2.2) to a second-order nonlinear equation

$$y_{zz}y_\tau - y_{z\tau}y_z = \frac{\mu^{-1}y_z F(\tau)}{s_0}. \tag{2.4}$$

The function $y(z, \tau)$ has a simple physical significance. Being the Lagrangian coordinate of the medium undergoing deformation, it determines which material point is located at the moment of time τ at point z [11].

We introduce the dimensionless variables and parameters:

$$\begin{gathered} Z = z/L,\ Y = y/L,\ t = \tau v_0/L,\ \Theta = (T - T_-)/(T_+ - T_-), \\ S = s/s_0,\ V = v/v_0,\ \eta = \mu_m/\mu,\ M(t) = F(t)L/v_0 s_0 \mu_m, \\ \delta = (b+c)/(c-b),\ \gamma = \delta + bc/a(c-b), \\ l = 1/\mu_0 \int\limits_{-1+\delta}^{1+\delta} \frac{1}{\mu\left[\frac{c-b}{2}x\right]}dx, \quad \beta = T_0/(T_+ - T_-), \quad \mu_m = \mu_0 e^\beta. \end{gathered} \tag{2.5}$$

Here, v_0 is the time-averaged feed, while $L = \mu_m \int\limits_b^c \frac{dx}{\mu(x)}$. With allowance for Eq. (2.5), Eq. (2.4) takes the form

$$Y_{ZZ}Y_t - Y_{Zt}Y_Z = M(t)\eta(Z)Y_Z. \qquad (2.6)$$

The use of dimensionless quantities leads to the equation

$$\int_{l(-1+\delta)}^{l(1+\delta)} \eta(s)\,ds = 1.$$

The dimensionless distributions of temperature and fluidity have the form

$$\Theta(Z) = \frac{[Z - l(1-\delta^2)/(\delta-\gamma)][Z - l(-1+\delta)][Z - l(1+\delta)][Z - l(1-\delta^2)/(\delta+\gamma)](\gamma^2-\delta^2)}{l^4(1-\delta^2)^3},$$

$$\eta(Z) = e^{\beta[1-1/\Theta(Z)]},\ l(-1+\delta) \leqslant Z \leqslant l(1+\delta),$$

where δ and γ are parameters accounting for the asymmetry and completeness of the temperature profile; β is a parameter characterizing the degree of heating of the glass in the deformation zone. Variation of the parameters δ, γ, and β makes it possible to obtain different distributions $\theta(Z)$ and $\eta(Z)$ in the deformation zone in order to select the optimum temperature regime. The dimensionless values of the velocity and cross section of the stream are calculated from the formulas

$$V = -Y_t/Y_Z,\ S = Y_Z. \qquad (2.7)$$

Let us examine the question of the boundary conditions for Eq. (2.6). The following functions of time are the initial data in the problem of fiber drawing: the inlet cross section S_-, the fed of the glass rod V_-, the drawing rate V_+. The last quantity is specified in order to determine the drawing force M(t), which is usually unknown under actual production conditions. The above functions must be augmented by one more function which characterizes the initial state of the fiber $Y_+(Z)$ at each point Z of the deformation zone. With allowance for this, we write the boundary conditions

$$\begin{gathered} Y(Z,\ 0) = Y_+(Z), \\ Y_Z[l(-1+\delta),\ t] = S_-(t), \\ Y_t[l(-1+\delta),\ t]/Y_Z[l(-1+\delta),\ t] = -V_-(t), \\ Y_t[l(1+\delta),\ t]/Y_Z[l(1+\delta),\ t] = -V_+(t). \end{gathered} \qquad (2.8)$$

The steady-state solution of problem (2.6), (2.8) for the case $S_- = 1$, $V_- = 1$, $V_+ = e^W$, where $W = \ln(V_+/V_-)$ has the form

$$Y_0 = (Z,\ t) = \int_{l(-1+\delta)}^{Z} e^{-W\xi(x)}dx - t,\quad \xi(x) = \int_{l(-1+\delta)}^{x} \eta(y)\,dy. \qquad (2.9)$$

Here, $Y_0(Z,\ t)$ depends on time but, in accordance with (2.7), gives physically observable quantities which are independent of time: rate and cross section.

A steady drawing process is optimum from a practical point of view. In such a process, the area of the outlet cross section of the fiber is constant. However, in reality, there are always random unsteady phenomena due to the presence of a large number of practically unavoidable perturbing factors. Thus, the feed of the glass rod and the drawing rate may experience small fluctuations connected with vibration of the unit or eccentricity of the pulleys and pinions in the drive mechanisms. The inlet cross section of the stream may change slightly due to geometric irregularities in the rod originating in its manufacture. Finally, the temperature – and thus the fluidity – of the molten glass is subject to fluctuations due to hydrodynamic instabilities during convective heat transfer in the heating zone and unsteady heat release by the heater or fluctuations of heat transfer in the cooling system of the heating element. The drawing itself may also turn out to be internally unstable, which would lead to an irregular intensification of small initial perturbations. All this indicates the need to analyze unsteady drawing processes.

We will examine two types of problems.

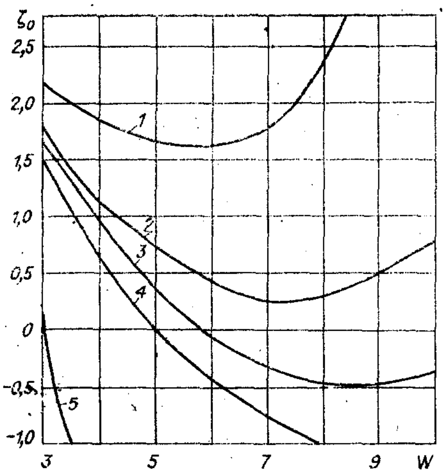

Fig. 1

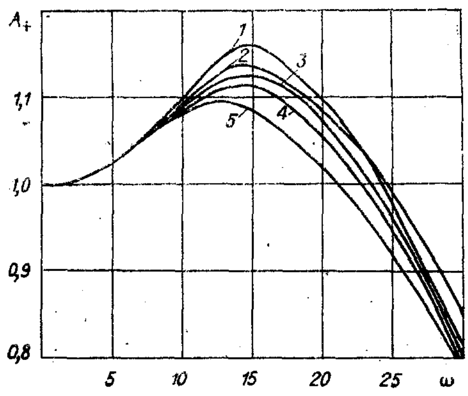

Fig. 2

1) Unsteady perturbations of a steady drawing regime caused by small fluctuations of regime parameters about their mean values (the problem of the reaction of the process to process-related interferences).

2) The stability of the drawing operation, examining the evolution, with constant regime parameters, of small perturbations of the initial state over time.

By virtue of the foregoing, we represent the boundary functions and fluidity as the sum of their mean values and small fluctuating complements:

$$V_- = 1 + \varepsilon\psi(t),\ V_+ = e^W + \varepsilon e^W \varkappa(t),\ S_- = 1 + \varepsilon\varphi(t), \qquad (2.10)$$
$$M = W + \varepsilon\alpha(t),\ \eta = e^{\beta[1-1/\Theta(Z)]} + \varepsilon\nu(Z,\ t).$$

Here, ψ, $\varkappa$, ϕ, and α are functions normalized on unity; ε is a dimensionless parameter which accounts for the smallness of the unsteady complements. This circumstance allows us to resort to standard linearization of the problem by the method of perturbation theory. To do this, the sought unsteady solution is represented in the form

$$Y(Z,\ t) = Y_0(Z,\ t) + \varepsilon P(Z,\ t), \qquad (2.11)$$

where P(Z, t) is an unsteady perturbation of motion.

Inserting (2.10) and (2.11) into (2.6), (2.8) and linearizing all of the relations with respect to ε, we have the boundary-value problem for P(Z, t):

$$e^{W\xi(Z)}P_{ZZ} + e^{W\xi(Z)}W\eta P_Z + P_{Zt} + W\eta P_t = -\alpha\eta - W\nu; \qquad (2.12)$$

$$P(Z,\ 0) = P_+(Z),\quad P_Z[l(-1+\delta),\ t] = \varphi(t), \qquad (2.13)$$
$$P_t[l(-1+\delta),\ t] = -\psi(t) - \varphi(t),\quad P_t[l(1+\delta),\ t]$$
$$= -e^W P_Z[l(1+\delta),\ t] - \varkappa(t).$$

Since any random fluctuations of the regime parameters can be represented as a superposition of elementary harmonics, we are mainly interested in the reaction of the drawing process to harmonic perturbations. As a result, we will examine only harmonic perturbations of the function having the form

$$\varphi(t) = \varphi_0 e^{-i\omega t},\ \psi(t) = \psi_0 e^{-i\omega t},\ \varkappa(t) = \varkappa_0 e^{-i\omega t}, \qquad (2.14)$$
$$\nu(Z,\ t) = \frac{\eta(Z)\,\nu_0}{\Theta^2(Z)}\cos\left[\frac{2k\Pi Z}{c-b} + n\right]e^{-i\omega t}.$$

From here, we write the solution of boundary-value problems of the type (2.12), (2.13) as

$$P(Z,\ t) = e^{\sigma t}F(Z),\quad \alpha = \alpha_0 e^{\sigma t}, \qquad (2.15)$$

where $F(Z) = P_1(Z) + iP_3(Z)$; $\alpha_0 = \alpha_1 + i\alpha_2$; $\sigma = -\zeta - i\omega$. (In problems concering the response to process interference, $\zeta = 0$.)

We introduce the notation:

$$P_2 = e^{W\xi(Z)}P_1', \quad P_4 = e^{W\xi(Z)}P_2'. \tag{2.16}$$

Inserting (2.14-2.16) into (2.12) and separating the real and imaginary parts, we obtain a system of ordinary differential equations in P_i (i = 1, 2, 3, 4):

$$\begin{aligned} P_1' &= e^{-W\xi}P_2, \\ P_2' &= -\alpha_1\eta - f(Z)W + W\eta\zeta P_1 + e^{-W\xi}\zeta P_2 - W\eta\omega P_3 - \omega e^{-W\xi}P_4, \\ P_3' &= e^{-W\xi}P_4, \\ P_4' &= -\alpha_2\eta + W\eta\omega P_1 + \omega e^{-W\xi}P_2 + W\eta\zeta P_3 + e^{-W\xi}\zeta P_4, \end{aligned} \tag{2.17}$$

where $f(Z) = \frac{\eta(Z)\nu_0}{\Theta^2(Z)}\cos\left[\frac{2k\Pi Z}{c-b} + n\right]$ for a problem on the response to temperature perturbations.

3. Let us examine a problem on the stability of the drawing operation. It is described mathematically by system (2.17) with f = 0 and the following homogeneous boundary conditions, obtained by insertion of (2.15) and (2.16) into (2.13) with zero perturbing functions ($\phi = \psi = \kappa = 0$):

$$P_1 = P_2 = P_3 = P_4 = 0, \quad Z = l[-1 + \delta]; \tag{3.1}$$

$$\omega P_3 + P_2 - \zeta P_1 = 0, \quad -\zeta P_3 - \omega P_1 + P_4 = 0, \quad Z = l[1 + \delta]. \tag{3.2}$$

To find a normal mode of the form (2.15) in the problem, it is necessary to calculate the roots ζ_0 and ω_0 of system (3.2). The values of the functions P_i (i = 1, 2, 3, 4) in this system at point $Z = \ell[1 + \delta]$, dependent on ζ and ω, are determined by solving (2.17) (with f = 0) with boundary conditions (3.1). Using the initial approximations of ζ^j and ω^j (j = 1, 2, 3) for fixed W, system (3.2) was solved numerically by Stephenson's generalized method. Meanwhile, Heming's method was used to solve problem (2.17), (3.1) at each step to calculate P_i with the current values of ζ and ω.

Figure 1 shows the dependence of the damping decrement ζ_0 of the lowest mode of problem (2.17), (3.1), (3.2) on the rate coefficient W for different temperature regimes and $\gamma = 10^{-7}$ (β = 1.2; 1.2; 1; 1.2; 10^{-7}, δ = −0.2; 0; 0; 0.2; 0 − lines 1-5). (The lowest mode is considered to be the mode with the smallest decrement [5].) The mode selection was determined by the selective of the initial approximation of ζ^i and ω^i. Stable regimes correspond to positive ζ_0, and the values of W at which $\zeta_0 = 0$ are called critical values (and designated as W_*). To illustrate the transition of the drawing operation from the stable regime to the unstable regime, we have chosen temperature regimes with small values of the parameter β at which this transition is seen. An actual drawing regime corresponds to values of β on the order of 10-20. As calculations showed, at these values the process is absolutely stable for practically any attainable value of W.

The most important result of the solution of this problem is the demonstration of the strong dependence of the stability of the drawing operation on the temperature regime and the possibility of stable, steady drawing in the nonisothermal regime at high values of the rate coefficient typical of optical-fiber drawing. Thus, this fact, already known from experience, receives theoretical substantiation.

4. In analyzing the reaction of the drawing operation to external disturbances, we examined four cases:

a) perturbations of drawing rate ($\varphi_0 = \psi_0 = \nu_0 = 0$, $\varkappa_0 = 1$) ;

b) perturbations of feed ($\varphi_0 = \varkappa_0 = \nu_0 = 0$, $\psi_0 = 1$) ;

c) perturbations of the inlet cross section ($\psi_0 = \varkappa_0 = \nu_0 = 0$, $\varphi_0 = 1$);

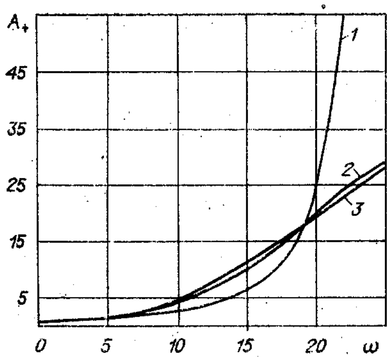

Fig. 3

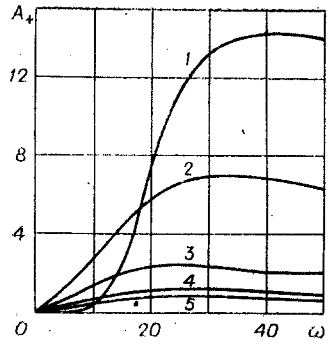

Fig. 4

d) perturbations of temperature and, thus, fluidity ($\varkappa_0 = \varphi_0 = \psi_0 = 0$, $\nu_0 = 1$).

The results of the solution are shown in the form of amplitude-frequency characteristics (AFC) describing the ratio of the relative amplitude of the perturbation of the outlet cross section of the fiber to the relative amplitude of the perturbation of the corresponding parameter:

$$A_+ = \frac{|\delta S(\omega)/S_+|}{|\delta a(\omega)/a|} = \sqrt{P_2^2[l(1+\delta)] + P_4^2[l(1+\delta)]},$$

where $S_+ = e^{-W}$ is the steady-state outlet cross section; $\delta a(\omega)$ and a are the perturbation and the steady-state value of the parameter.

Figure 2 ($\gamma = 10^{-7}$) shows the AFC for perturbations of drawing rate or feed* with a fixed value of $W = \ln 10^4$ for different stable temperature regimes (δ = 0; 0.1; 0; 0.1; –0.2, β = 5; 10; 10; 10; 10 – lines 1-5). It follows from them that the reaction of the process rapidly decays with an increase in frequency and becomes insignificant for values of dimensionless frequency $\omega \gtrsim 50$, which corresponds to $\sim$0.1 Hz. A small increase in the relative perturbations of the outlet section of the fiber in the low-frequency region does not lead to a gain greater than 1.2 in any of the temperature regimes and is quickly replaced by a sharp decrease after attainment of a maximum at frequencies $\omega \sim 15$. The main factors affecting the reaction of the process in the case of drawing-rate perturbations is the degree of heating of the glass mass (the parameter β) and the degree and direction of asymmetry of the temperature profile (the parameter δ). The optimum temperature regime from the viewpoint of stability is the regime characterized by the presence of a temperature maximum in the bottom region of the deformation zone. The drawing operation also stabilizes with a decrease in the temperature T_+.

Figure 3 (β = 1.5; 5; 10 – lines 1-3, $\delta = 0$, $\gamma = 10^{-7}$) shows the AFC for perturbations of the inlet section. Here, the main difference from the previous case is a continuous increase in the AFC with frequency and a gain which is more than order of magnitude greater. Nevertheless, perturbations of the inlet cross section cannot lead to significant fluctuations in the outlet diameter of the fiber in the high-frequency region because high frequencies of oscillation of the inlet section correspond to a small spatial period of the geometric irregularities of the rod and a low relative amplitude for these irregularities. Thus, dimensionless frequencies $\omega \sim 30$ or more correspond to a spatial period of 1 mm or less for the irregularities. It is clear that a glass rod 1 cm in diameter – for which the final processing operation is fire polishing – cannot have significant geometric irregularities on such a small scale.

The dependence of the AFC on the regime temperature parameters is unimportant except for the parameter β (curve 1, Fig. 3). However, β may vary broadly ($\beta \sim$ 5-15) for actual conditions of drawing quartz fiber. The results of calculation of the AFC for perturbations of the fluidity of the molten glass are shown in Fig. 4 (β = 10; 10; 5; 10; 10; k = 10; 1; 1; 1; 0, ϕ = 0; $\pi/2$; 0; 0; 0 – lines 1-5, $\delta = 0$, $\gamma = 10^{-7}$). This case is characterized by higher gains (tenfold). Also seen here is a maximum of the reaction of the process to the perturbations at frequencies $\omega \sim$ 30-40. However, further decay of the reaction with an increase in

*The numerical experiment showed that the reaction of the drawing operation to perturbations of these parameters is the same.

frequency occurs very slowly, so that the reaction remains significant for high frequencies as well. With allowance for the fact that perturbations of temperature and, thus, fluidity are characterized by relatively high frequencies (on the order of 1 Hz or $\omega \sim 500$ in dimensionless form), the last result leads to the conclusion that temperature perturbations in the deformation zone are the main cause of high-frequency perturbations of the outlet cross section of the fiber.

It is important to note that we observed a sharp increase in the reaction of the process with a decrease in the spatial scale of the temperature perturbations. The AFC's obtained and typical values of the dimensional parameters produce the following estimate of the effect of temperature perturbations: for temperature perturbations with an amplitude of 1 deg C at frequencies greater than 0.1 Hz, the amplitude of the perturbation of the diameter of an optical fiber may be 3 μm.

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